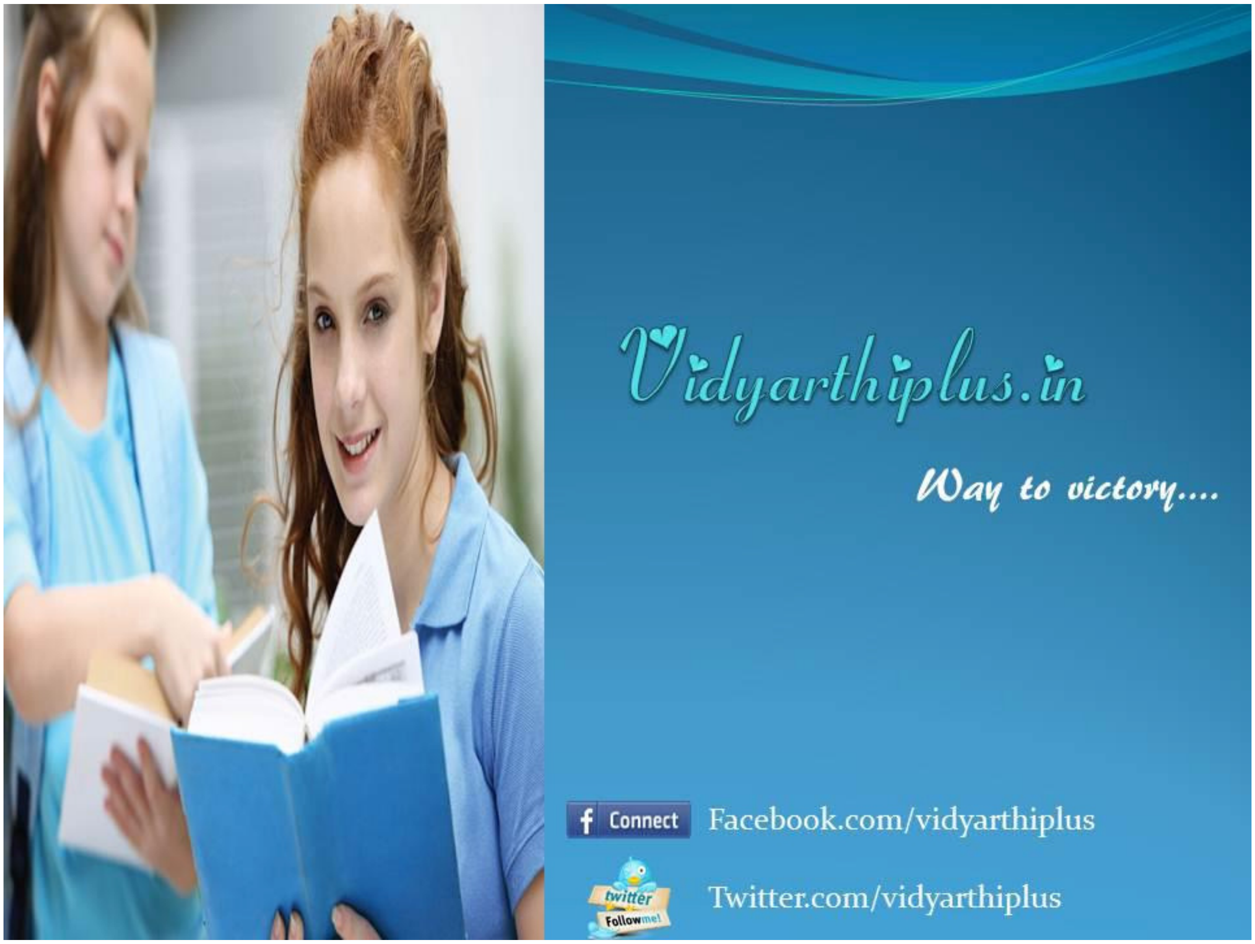

Mathematics – II – Some Important Questions

Part A

1. Find the particular integral of $y'' - 3y' + 2y = e^x - e^{2x}$.
2. Solve $x^2 y'' - 2xy' + 2y = 0$.
3. Eliminate y from the system $\frac{dx}{dt} + 2y = -\sin t, \frac{dy}{dt} - 2x = \cos t.$
4. What is the greatest rate of increase of $\phi = xyz^2$ at (1, 0, 3).
5. Find ‘a’ such that $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.
6. If $\vec{F} = (4xy - 3x^2 z^2)\vec{i} + 2x^2 \vec{j} - 2x^3 z\vec{k}$. Check whether the integral $\int_c \vec{F}.d\vec{r}$ is independent of the path c.
7. Check the analyticity of the function $2xy + i(x^2 - y^2)$.

8. Find the constant 'a' so that $u(x,y) = ax^2 - y^2 + xy$ is harmonic.
9. Find the image of $2x + y - 3 = 0$ under the transformation $w = z + 2i$.
10. Obtain the invariant points of the transformation $w = 2 - \frac{2}{z}$.

1. Find the particular integral of $y'' - 4y = 3^x$..
2. Solve $x^2 y'' - xy' + y = 0$..
3. Eliminate x from the system $\frac{dy}{dt} + 2x = 0, \frac{dx}{dt} - 2y = 0.$
4. Find the area of a circle of radius 'a' using Green's theorem.
5. Find 'a' such that $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.
6. If $\vec{F} = (4xy - 3x^2 z^2)\vec{i} + 2x^2 \vec{j} - 2x^3 z\vec{k}$. Check whether the integral $\int_c \vec{F}.d\vec{r}$ is independent of the path c.
7. Check the analyticity of the function $2xy + i(x^2 - y^2)$.
8. Find the constant 'a' so that $u(x,y) = ax^2 - y^2 + xy$ is harmonic.
9. Find the image of $2x + y - 3 = 0$ under the transformation $w = z + 2i$.
10. Obtain the invariant points of the transformation $w = 2 - \frac{2}{z}$.
11. State Cauchy's Residue theorem.
12. Obtain the residues of the function $f(z) = \frac{z - 3}{(z+1)(z+2)}$ at its poles.
13. Find the inverse L.T. of $\frac{1}{s(s-5)}$
14. If L[f(t)] = F(s) , P.T. L [f(3t)] = $\frac{1}{3}F\left(\frac{s}{3}\right)$.

16 marks

1.(a) (i) Solve $(D^2 - 2D + 5)y = e^{2x}\sin x + 2$.

(ii) Solve $(D^2 - 4D + 4)y = e^{2x}$ by the method of variation of parameters.

(b) (i) Solve the simultaneous equations

$$\frac{dx}{dt} + 2x + 3y = 2e^{2t} \quad ; \frac{dy}{dt} + 3x + 2y = 0.$$

(ii) Solve $\left[(x+1)^2 D^2 + (x+1)D + 1\right]y = 4\cos[\log(x+1)]$.

2.(a) (i) Find 'a' and 'b' such that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2 y + z^3 = 4$ cut orthogonally at (1, -1, 2).

(ii) Show that $r^n \vec{r}$ is an irrotational vector for any value of n but it is solenoidal only if n = -3.

(b) (i) Using Green's theorem evaluate $\int_c (y - \sin x)dx + \cos x\, dy$ where c is the triangle OAB where O = (0, 0), $A = \left(\frac{\pi}{2}, 0\right), B = \left(\frac{\pi}{2}, 1\right)$.

(ii) Prove $\vec{F} = \left(y^2 \cos x + z^3\right)\vec{i} + (2y \sin x - 4)\vec{j} + 3xz^2\vec{k}$ is irrotational and find its scalar potential.

3.(a) (i) Verify stroke's theorem for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ over the open surfaces of the cube $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$ not included in the xoy plane.

(ii) Find the work done, when a force $\vec{F} = \left(x^2 - y^2 + x\right)\vec{i} - (2xy + y)\vec{j}$ moves a particle from the origin to the point (1, 1) along $y^2 = x$.

(b) (i) Verify Gauss divergence theorem for $\vec{F} = \left(x^2 - yz\right)\vec{i} + \left(y^2 - zx\right)\vec{j} + \left(z^2 - xy\right)\vec{k}$ and S is the surface of the rectangular parallelepiped bounded by x = 0, x = a, y = 0, y = b, z = 0, z = c .

(ii) Prove $div\ (u\ \ grad\ v) = u\nabla^2 v + (grad\ u).(grad\ v)$.

4.(a) (i) If $f(z) = u + iv$ is a regular function of z in a domain D the following relations hold in D.

$\nabla^2\left[|f(z)|^2\right] = 4|f'(z)|^2$.

(ii) If $w = f(z)$ is analytic prove that $\frac{dw}{dz} = \frac{\partial w}{\partial x} = -i\frac{\partial w}{\partial y}$ where z = x+iy And prove that

$\frac{\partial^2 w}{\partial z \partial \bar{z}} = 0$.

(b) (i) Determine the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$.

(ii) Show that $\frac{x}{x^2 + y^2}$ is harmonic.

5.(a) (i) Prove that an analytic function with constant modulus is constant.

(ii) Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$.

(b) (i) Find the bilinear transformation which maps the points $1, i, -1$ onto the points $0, 1, \infty$. Show that the transformation maps the interior of the unit circle of the z-plane onto the upper half of the w-plane.

(ii) Find the image of the circle $|z| = 2$ under the transformation $w = z + 3 + 2i$.

6 (a) (i) Solve $\left(D^2 - 2D + 5\right)y = e^{2x}\sin x + 2$.

(ii) Solve $\left(D^2+4\right)y=\sec 2x$ by the method of variation of parameters.

(b) (i) Solve the simultaneous equations

$$\frac{dx}{dt}+2x+3y=2e^{2t} \quad ; \frac{dy}{dt}+3x+2y=0.$$

(ii) Solve $\left[(x+2)^2D^2-(x+2)D+1\right]y=x+2$.

7 .(a) (i) Find 'a' and 'b' such that the surfaces $ax^2-byz=(a+2)x$ and $4x^2y+z^3=4$ cut orthogonally at (1, -1, 2).

(ii) Show that $r^n\vec{r}$ is an irrotational vector for any value of n but it is solenoidal only if n = -3.

(b) (i) Using Green's theorem evaluate $\int_c (y-\sin x)dx+\cos x\,dy$ where c is the triangle OAB where O = (0, 0), $A=\left(\frac{\pi}{2},0\right), B=\left(\frac{\pi}{2},1\right)$.

(ii) Prove $\vec{F}=\left(y^2\cos x+z^3\right)\vec{i}+\left(2y\sin x-4\right)\vec{j}+3xz^2\vec{k}$ is irrotational and find its scalar potential.

8.(a) (i) Verify stroke's theorem for $\vec{F}=(x^2-y^2)\vec{i}+2xy\vec{j}$ over the rectangular region in the xoy plane bounded by the lines $x=0, y=0, x=a, y=b.$

(ii) Find the work done, when a force $\vec{F}=\left(x^2-y^2+x\right)\vec{i}-(2xy+y)\vec{j}$ moves a particle from the origin to the point (1, 1) along $y^2=x$.

(b) (i) Verify Gauss divergence theorem for $\vec{F}=\left(x^2-yz\right)\vec{i}+\left(y^2-zx\right)\vec{j}+\left(z^2-xy\right)\vec{k}$ and S is the surface of the rectangular parallelepiped bounded by x = 0, x = a, y = 0, y = b, z = 0, z = c .

(ii) Prove $div\ (u\ \ grad\ v)=u\nabla^2 v+\left(grad\ u\right).\left(grad\ v\right).$

9 .(a) (i) If $f(z)=u+iv$ is a regular function of z in a domain D the following relations hold in D.

$$\nabla^2\left[\left|f(z)\right|^2\right]=4\left|f'(z)\right|^2.$$

(ii) If $w=f(z)$ is analytic prove that $\frac{dw}{dz}=\frac{\partial w}{\partial x}=-i\frac{\partial w}{\partial y}$ where z = x+iy And prove that

$$\frac{\partial^2 w}{\partial z\partial\bar{z}}=0.$$

(b) (i) Determine the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y-\cos 2x}$.

(ii) Show that $\frac{x}{x^2+y^2}$ is harmonic.

10 .(a) (i) Prove that an analytic function with constant modulus is constant.

(ii) Find the image of $|z-2i|=2$ under the transformation $w=\frac{1}{z}$.

(b) (i) Find the bilinear transformation which maps the points -1,0,1 in the Z plane onto the points 0,i,3i in the w plane.

(ii) Find the image of the circle $|z|=2$ under the transformation $w=z+3+2i$.

11.Using Cauchy's integral formula evaluate $\int_c \frac{zdz}{(z-1)(z-2)}$ where C is the circle $|z-2|=1/2$.

(ii)Find the Laurent's series expansion for the function $f(z)=\frac{7z-2}{z(z-2)(z+1)}$ in the region $1<|z+1|<3$.

b) (i)Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5S\text{in}\,\theta}$ using contour integration.

12. . Find L.T. of $t^2e^{2t}\cos 2t$

(ii) Find L.T.of f(t) if f(t) = e^t ,0 < t < 2 π with $f(t)=f(t+2\pi)$.

b) (i)Solve using Laplace Transform $\frac{d^2y}{dt^2}+4\frac{dt}{dt}+8y=1$ given y(0)=0 and y'(0)= -1.

(ii) Using convolution theorem find the inverse Laplace transform of $\frac{2}{(s+1)(s^2+4)}$