

Assignment #1MATRICESI PART-A (Short Answer Questions)

1. Define eigen value and eigen vector of a matrix.
2. Prove that A and A^T have the same eigen values.
3. Find the eigen values of $2A^2$, if $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.
4. Find the sum and product of the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & -3 \\ -2 & -1 & -3 \end{bmatrix}$.
5. Two eigen values of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to 1 each. Find the third eigen value.
6. Find the eigen vectors of $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$.
7. The product of two eigen values of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigen value.
8. State Cayley-Hamilton theorem.
9. Give two applications of Cayley-Hamilton theorem.
10. What do you mean by diagonalizing a matrix?
11. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$.
12. Use Cayley-Hamilton theorem to find the inverse of $A = \begin{bmatrix} 7 & 3 \\ 2 & 6 \end{bmatrix}$.

13. Define a quadratic form and give an example for the same in three variables.
14. Write down the quadratic form corresponding to the matrix $\begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 3 \end{bmatrix}$.
15. Write down the matrix of the quadratic form $3x_1^2 + 5x_2^2 + 5x_3^2 - 2x_1x_2 + 2x_2x_3 + 6x_3x_1$.
16. When is a quadratic form said to be singular?
17. What do you mean by canonical form of a quadratic form?
18. Define index and signature of a quadratic form.
19. Find the index and signature of the quadratic form $x_1^2 + 2x_2^2 - 3x_3^2$.
20. Write down the nature of quadratic form.

PART-B

Find the eigen values and eigen vectors of the following matrices:

21. $\begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$

22. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

23. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$ and hence find A^{-1} .

24. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ and hence find A^4 .

25. Find A^n , using Cayley-Hamilton theorem, when $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$. Hence find A^4 .

26. Diagonalize the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ by similarity transformation.

27. Diagonalize the matrix $B = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ by orthogonal transformation.

28. Reduce the quadratic form

$$3x_1^2 - 3x_2^2 - 5x_3^2 - 2x_1x_2 - 6x_2x_3 - 6x_3x_1$$

to canonical form by an orthogonal transformation. Also find the rank, index and signature of the quadratic form.

29. Reduce the quadratic form

$$10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_2$$

to a canonical form by orthogonal reduction.

Find also a set of non-zero values of x_1, x_2, x_3 , which will make the quadratic form zero.

30. Determine the nature of the following quadratic form without reducing them to canonical form:

$$x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_2x_3 - 2x_3x_1.$$